5/H-29 (vi) (Syllabus–2015)

2017

(October)

MATHEMATICS

(Honours)

(Differential Equations and Advanced Dynamics)

(GHS–52)

Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer Differential Equations and Advanced Dynamics in separate books

Answer **five** questions, choosing **one** from each Unit

UNIT—I

1. *(a)* Solve the differential equation 6

$$x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x+\log x$$

(Turn Over)

(b) Solve the equation

$$(1+x+x^2)\frac{d^3y}{dx^3}+(3+6x)\frac{d^2y}{dx^2}+6\frac{dy}{dx}=0$$ 6

(c) Solve the following equation : 3

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=e^{5x}$$

2. (a) Apply the method of variation of parameters to solve the equation

$$\frac{d^2y}{dx^2}+k^2y=\sec kx$$ 6

(b) Solve the following simultaneous differential equations : 6

$$\frac{dx}{dt}+4x+3y=t$$

$$\frac{dy}{dt}+2x+5y=e^t$$

(c) Solve the following equation : 3

$$\frac{dx}{z(x+y)}=\frac{dy}{z(x-y)}=\frac{dz}{x^2+y^2}$$

(Continued)

UNIT—II

(In this unit, $p=\frac{\partial z}{\partial x}$, $q=\frac{\partial z}{\partial y}$)

3. (a) Form a partial differential equation by eliminating a, b, c from

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ 6

(b) Solve the following equation : 3

$$\frac{y^2zp}{x}+zxq=y^2$$

(c) Apply Charpit's method to find the complete integral of

$$(p^2+q^2)y=qz$$ 6

4. (a) Find the integral surface of the partial differential equation $(x-y)p+(y-x-z)q=z$ through the circle $z=1$, $x^2+y^2=1$. 6

(b) Find the complete integral and singular integral of

$$z=px+qy+c\sqrt{1+p^2+q^2}$$ 6

(c) Find the complete integral of $p^2=zq$. 3

(Turn Over)

UNIT—III

5. Establish the formula

$$\frac{d^2u}{d\theta^2}+u=\frac{P}{h^2u^2},\ \dot{\theta}=hu^2$$

where $u=\frac{1}{r}$ for the motion of a particle describing a central orbit under an attraction P per unit mass.

If $P=\mu u^5$, find the speed v with which the particle can describe the circle $r=a$.

If the particle moves under this attraction with the same areal constant as the circular path and

$$\dot{r}=-\frac{3v}{4\sqrt{2}},\ \text{where } r=2a,\ \theta=0$$

find the equation of the spiral path of the particle and show that as $\theta\to\infty$, the path is asymptotic to the circle $r=a$. 6+2+7=15

6. (a) A particle slides in a verticle plane down a rough cycloidal arc whose axis is vertical and vertex downwards, starting from a point, where the tangent makes an angle θ with the horizon and coming to rest at the vertex. Show that $\mu e^{\mu\theta}=\sin\theta-\mu\cos\theta$, μ being the coefficient of friction. 8

(Continued)

(b) If $P=\mu(u^2-au^3)$, where $a>0$ and a particle is projected from an apse at a distance a from the centre of force with a velocity $\sqrt{\mu c/a^2}$, where $a>c$, then prove that the other apsidal distance of the orbit is $\frac{a(a+c)}{a-c}$. 7

UNIT—IV

7. (a) For a coplanar rigid system, prove that the principal moments of inertia at a point are the extreme values of the moments of inertia at that point. Show further that these extreme values are given by

$$I_{\min}=\frac{1}{2}\left[A+B-\sqrt{(B-A)^2+4F^2}\right]$$

$$I_{\max}=\frac{1}{2}\left[A+B+\sqrt{(B-A)^2+4F^2}\right]$$

where A, B and F have their usual meanings. 8

(b) A uniform rigid rod AB moves so that A and B have velocities $\vec{U}_A$ and $\vec{U}_B$ at any instant. Show that the kinetic energy is then

$$T=\frac{1}{6}M\left[\vec{U}_A^2+\vec{U}_A.\vec{U}_B+\vec{U}_B^2\right]$$

where M is the mass of the rod. 7

(Turn Over)

8. (a) A uniform solid rectangular block is of mass M and have dimensions $2a$, $2b$, $2c$. Find the equation of the momental ellipsoid for a corner O of the block, referred to edges through O as coordinate axes. Determine the moment of inertia about OO', where O' is the point diagonally opposite to O. 10

(b) Find the moment of inertia of a rectangular lamina of sides $2a$, $2b$ about a line parallel to one of its sides. 5

UNIT—V

9. (a) A uniform rod AB of mass $2m$ is freely jointed at B to a second rod BC of mass m. The rods lie on a smooth horizontal plane at right angles to each other and an impulse I is applied to AB at A in a direction parallel to BC. Find the initial velocity of BC and prove that the kinetic energy generated is $\frac{5}{6}I^2/m$. 6+2=8

(b) A circular hoop of radius a, rotating in a vertical plane with spin ω and with its centre at rest, is in contact with a rough plane inclined at an angle α, the angle of friction for the surfaces in contact also being α. Show that, if the initial slip velocity is down the plane, the hoop

(Continued)

remains stationary for the time $\frac{a\omega}{g\sin\alpha}$ and the hoop rolls down the plane with acceleration $\frac{1}{2}g\sin\alpha$. 7

10. (a) A uniform rod is placed on a horizontal table with two-thirds of its length hanging over the edge of the table. If the rod is at right angles to the edge and is released, show that it will begin to slip when the rod has turned through an angle of

$$\tan^{-1}\left(\frac{1}{2}\mu\right)$$

where μ is the coefficient of friction between rod and table. 7

(b) A uniform circular cylinder of mass M and radius a rolls down a rough inclined plane, inclined at an angle α to the horizontal. Prove that its velocity down the plane is given by

$$v = \frac{2}{3}gt\sin\alpha + v_0$$

and that its angular velocity at time t is given by

$$\omega = \frac{2gt\sin\alpha}{3a} + \omega_0$$

where v_0 and ω_0 are the initial values of v and ω respectively at time $t = 0$. 8

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